38

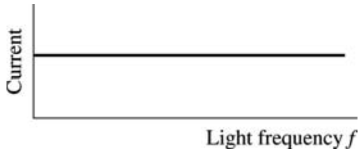
QUANTIZATION

Conceptual Questions

38.1. (a) The ultraviolet light consists of bundles of energy that are absorbed by the electrons on the electroscope. The frequency of these bundles of energy is enough to allow the escape of electrons from the surface of the electroscope.
(b) The ultraviolet light does, in fact, cause a few electrons to be emitted from an uncharged electroscope. But once the electroscope has acquired even a *very tiny* positive charge, further photoelectrons (negatively charged) cannot escape the electric force that pulls them back to the positive electroscope. This is like projectiles being launched from the earth's surface with less than the escape speed. The very small positive charge that the electroscope is able to acquire is not enough to deflect the leaves and thus has no noticeable effect.

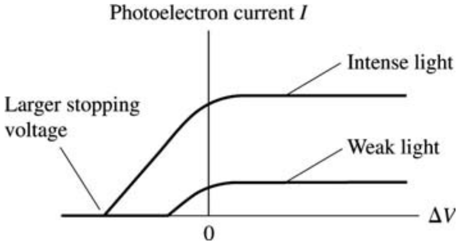
38.2. (a) When $\Delta V > 0$, all the emitted electrons are attracted to and collected by the anode. This means a further increase in the voltage cannot change the number of electrons arriving per second and thus cannot increase the current.
(b) The work function E_0 is the *minimum* energy an electron needs to escape from the metal. Some electrons, such as those a bit farther from the surface, need more than E_0 to escape. There is a *range* of escape energies, so the escaped electrons have a range of kinetic energies and not a single kinetic energy.
(c) If the anode potential is V, an electron leaving the cathode with kinetic energy K arrives at the anode with kinetic energy $K' = K + eV$. A negative V causes a decrease in kinetic energy. K' *cannot* become negative, so for $eV \le -K$ the electron is repelled by the anode and turned back toward the cathode. The emitted electrons have a maximum kinetic energy K_{max}. When $eV = -K_{\text{max}}$, *all* electrons are turned back and the current drops to zero. If the current reaches zero at $V = -V_{\text{stop}}$, then $V_{\text{stop}} = K_{\text{max}}/e$. The stopping voltage measures the maximum kinetic energy by causing the most energetic electrons, those with $K = K_{\text{max}}$, to be turned back from the anode.

38.3.



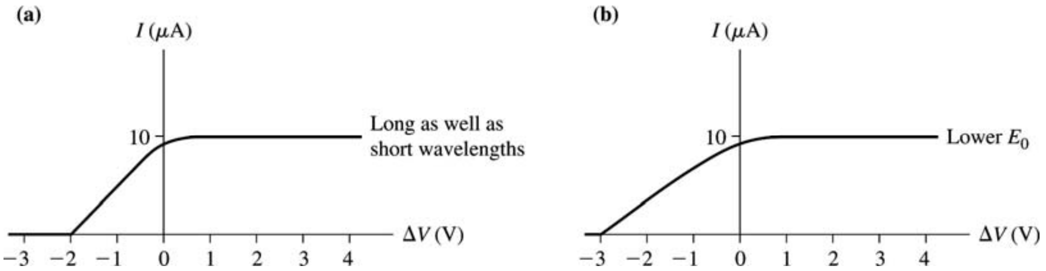
According to classical physics, the photoelectric current is *not* dependent on the light's frequency. If the light intensity remains constant (same energy per second falling on the metal), the photoelectric current should be constant with *no threshold* as the frequency is changed. Thus, the graph would be a horizontal line, starting from $f = 0$ Hz.

38.4.



The graphs would still be horizontal for $\Delta V > 0$ V because all the electrons are being collected. Also, more intense light would still give a larger current. But classical physics postulates a *thermal* cause for the photoelectric effect. If this were true, more intense light would heat the electrons more and thus eject the electrons with more kinetic energy. In this case, more intense light would have a *larger stopping voltage*. This would appear on the graph as a more negative *x*-intercept.

38.5. The photoelectric current depends on the potential difference ΔV between the two electrodes, the nature of the cathode metal, and the intensity of the light.



Solve: **(a)** According to classical physics, there is no dependence on the light's wavelength. If the light intensity remains constant (same amount of energy falling on the metal cathode), the photocurrent will be unchanged.
(b) The maximum kinetic energy of the electrons emitted from a cathode is $K_{\text{max}} = E_{\text{elec}} - E_0$. If E_0 is smaller for a different metal, the emitted electrons will have a higher kinetic energy and thus the stopping potential will be larger.

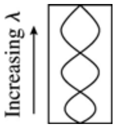
38.6. Lower speed. Since $\lambda_1 = \lambda_2$ then $f_1 = f_2$. And $(E_{\text{elec}})_1 = hf_1$ is equal to $(E_{\text{elec}})_2 = hf_2$. But $(E_0)_1 > (E_0)_2$; $(K_{\text{max}})_1 = E_{\text{elec}} - (E_0)_1$ and $(K_{\text{max}})_2 = E_{\text{elec}} - (E_0)_2$, so $(K_{\text{max}})_1 < (K_{\text{max}})_2$ and $v_1 < v_2$.

38.7. Electron 1. $K_1 = 100$ eV and $K_2 = 200$ eV. $v = \sqrt{\frac{2K}{m}}$. Therefore $v_2 > v_1$. $\lambda = \frac{h}{mv}$ so $\lambda_1 > \lambda_2$.

38.8. The electron. With $\lambda = \frac{h}{mv}$ and $v = \sqrt{\frac{2K}{m}}$ then $\lambda = \frac{h}{m\sqrt{\frac{2K}{m}}} = \frac{h}{\sqrt{2Km}}$. So the particle with the smaller mass has the larger wavelength.

38.9. **(a)** The number of dots will increase, but will still appear in bands of the same width. **(b)** When the speed is reduced, the wavelength increases, leading to wider, more spread out fringes with the same number of dots in each fringe as originally. **(c)** Neutrons have more mass than electrons, and therefore more momentum at the same speed. More momentum means a smaller wavelength, so the fringes are narrower and closer together. **(d)** The pattern becomes a single slit diffraction pattern. The two-slit interference pattern is eliminated.

38.10. The particle has to set up a standing wave inside the box. An $n = 3$ wave has three antinodes. In this case, however, the particle's wavelength is not constant. The particle's de Broglie wavelength is longer near the top of the box, where its velocity is slower, and shorter near the bottom of the box, where its velocity is faster. The spacing between two adjacent nodes on a standing wave is $\lambda/2$, so the nodes will be farther apart near the top of the box and closer together near the bottom. This leads to a standing wave like the one shown in the figure.



38.11. No, the electron is not at rest. It is in an allowed orbit. The (quantized) orbits that are allowed are called stationary states.

38.12. **(a)** $|E_1| = 4.0\ \text{eV}$

(b) $E_{\text{photon}} = 3.0\ \text{eV}$ because it had to go from state $n = 1$ to state $n = 3$ to then emit the 1240 nm photon.

(c) $E_{\text{elec}} = 3.0\ \text{eV}$ for the same reasons.

Exercises and Problems

Exercises

Section 38.1 The Photoelectric Effect

Section 38.2 Einstein's Explanation

38.1. Model: Light of frequency f consists of discrete quanta, each of energy $E = hf$.

Solve: (a) The energy of the light quantum is

$$E = hf = h\frac{c}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J s})(3.0\times10^{8}\ \text{m/s})}{400\times10^{-9}\ \text{m}} \times \frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}} = 3.11\ \text{eV}$$

From Table 38.1, the work functions for sodium and potassium are smaller than 3.11 eV. That is, light of wavelength 400 nm has enough energy to eject photoelectrons from sodium and potassium.

(b) The energy of the light quantum is

$$E = hf = h\frac{c}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J s})(3.0\times10^{8}\ \text{m/s})}{250\times10^{-9}\ \text{m}} \times \frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}} = 4.97\ \text{eV}$$

Light of wavelength 250 nm has enough energy to eject photoelectrons from all of the metals on the table except gold.

38.2. Solve: A steady photoelectric current of $10\,\mu\text{A}$ is indicated in the graph. The number of electrons per second is

$$10\ \mu\text{A} = 10\ \frac{\mu\text{C}}{\text{s}} = 1.0\times10^{-5}\,\frac{\text{C}}{\text{s}} \times \frac{1\ \text{electron}}{1.6\times10^{-19}\ \text{C}} = 6.25\times10^{13}\ \text{electrons/s}$$

38.3. Solve: From Equation 38.7, the maximum kinetic energy is

$$K_{\max} = hf - E_0 = h\frac{c}{\lambda} - E_0$$

$$\Rightarrow \lambda = \frac{hc}{E_0 + K_{\max}} = \frac{(6.63\times10^{-34}\ \text{J s})(3.0\times10^{8}\ \text{m/s})}{4.28\ \text{eV} + 1.30\ \text{eV}} \times \frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}} = 223\ \text{nm}$$

Assess: $\lambda = 223\ \text{nm}$ is the wavelength of light in the ultraviolet region of the spectrum.

38.4. Model: Light of frequency *f* consists of discrete quanta, each of energy $E = hf$.

Solve: The lowest photon energy that creates photoelectrons from the metal is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J s})(3.0\times10^{8}\text{ m/s})}{388\times10^{-9}\text{ m}}\times\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}} = 3.20\text{ eV}$$

The work function of the metal is $E_0 = 3.20$ eV.

38.5. Model: The threshold frequency for the ejection of photoelectrons is $f_0 = E_0/h$ where E_0 is the work function.

Solve: The visible region of light extends from 400 nm to 700 nm. For $\lambda_0 = 400$ nm, the work function is

$$E_0 = f_0 h = \frac{hc}{\lambda_0} = \frac{(6.63\times10^{-34}\text{ J s})(3.0\times10^{8}\text{ m/s})}{400\times10^{-9}\text{ m}}\times\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}} = 3.11\text{ eV}$$

For $\lambda_0 = 700$ nm,

$$E_0 = \frac{(6.63\times10^{-34}\text{ J s})(3.0\times10^{8}\text{ m/s})}{700\times10^{-9}\text{ m}}\times\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}} = 1.78\text{ eV}$$

The cathode that will work in the entire visible range must have a work function of 1.78 eV or less.

38.6. Solve: (a) A metal can be identified by its work function. From Equation 38.8, the stopping potential is

$$V_{\text{stop}} = \frac{hf - E_0}{e} \Rightarrow E_0 = hf - eV_{\text{stop}}$$

The frequency and energy of the photons are

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{200\times10^{-9}\text{ m}} = 1.500\times10^{15}\text{ Hz} \Rightarrow hf = (4.14\times10^{-15}\text{ eV s})(1.500\times10^{15}\text{ Hz}) = 6.21\text{ eV}$$

If the stopping potential is $V_{\text{stop}} = 1.56$ V, then $eV_{\text{stop}} = 1.56$ eV. Thus,

$$E_0 = hf - eV_{\text{stop}} = 6.21\text{ eV} - 1.56\text{ eV} = 4.65\text{ eV}$$

Using Table 38.1, we can identify the metal as *copper*.

(b) The kinetic energy of the electrons and thus the stopping potential are *independent* of the light intensity. A more intense light generates more electrons, but the electrons still have the same kinetic energy. The stopping potential is $V_{\text{stop}} = 1.56$ V after the intensity is doubled.

Section 38.3 Photons

38.7. Solve: (a) Using Equation 38.4, the wavelength of the photon is

$$\lambda = \frac{c}{f} = \frac{c}{E/h} = \frac{hc}{E} = \frac{(4.14\times10^{-15}\text{ eV s})(3\times10^{8}\text{ m/s})}{0.30\text{ eV}} = 4.14\times10^{-6}\text{ m} = 4140\text{ nm}$$

This is infrared light.

(b) Likewise, for an energy of 3.0 eV, the wavelength is $\lambda = 414$ nm and is in the visible region.

(c) For an energy of 30.0 eV, the wavelength is $\lambda = 41.4$ nm and is in the ultraviolet region.

Assess: Since $E \propto \lambda^{-1}$, the higher the energy of the photon, the smaller its wavelength.

38.8. Solve: (a) The frequency of the photon is

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{550\times10^{-9}\text{ m}} = 5.45\times10^{14}\text{ Hz}$$

From Equation 38.4, the energy is

$$E = hf = (4.14\times10^{-15}\text{ eV s})(4.29\times10^{14}\text{ Hz}) = 2.26\text{ eV}$$

(b) The frequency of the photon is

$$f = \frac{E}{h} = \frac{7500\ \text{eV}}{4.14\times10^{-15}\ \text{eV s}} = 1.812\times10^{18}\ \text{Hz}$$

Thus, the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{1.208\times10^{18}\ \text{Hz}} = 1.656\times10^{-10}\ \text{m} = 0.166\ \text{nm}$$

Assess: Because x-ray photons are very energetic, their wavelength is small.

38.9. Solve: (a) From Equation 38.4, the energy of the radio-frequency photon is

$$E = hf = (4.14\times10^{-15}\ \text{eV s})(450\times10^{6}\ \text{Hz}) = 1.86\times10^{-6}\ \text{eV}$$

(b) The energy of the visible light photon

$$E = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15}\ \text{eV s})(3.0\times10^{8}\ \text{m/s})}{450\times10^{-9}\ \text{m}} = 2.76\ \text{eV}$$

(c) The energy of the x-ray photon is

$$E = \frac{hc}{\lambda} = \frac{(4.14\times10^{-15}\ \text{eV s})(3.0\times10^{8}\ \text{m/s})}{0.045\times10^{-9}\ \text{m}} = 27.6\ \text{keV}$$

38.10. Solve: (a) From Equation 38.4, the energy of each photon is

$$E_{\text{photon}} = hf = (6.63\times10^{-34}\ \text{J s})(101\times10^{6}\ \text{Hz}) = 6.696\times10^{-28}\ \text{J}$$

The number of photons in $10^4\ \text{J}$ is

$$N = \frac{E_{\text{total}}}{E_{\text{photon}}} = \frac{10^{4}\ \text{J}}{6.696\times10^{-28}\ \text{J}} = 1.5\times10^{29}$$

The antenna emits 1.5×10^{29} photons per second.
(b) The number of photons emitted per second is so enormous that we couldn't possibly recognize the effects of single photons. It's safe to treat the broadcast as an electromagnetic wave.

38.11. Solve: The rate of photon emission by a laser is the number of photons emitted per second. If *P* is the power of the beam, the rate is

$$R_{\text{photon}} = \frac{P}{hf} = \frac{P}{hc/\lambda} = \frac{P\lambda}{hc}$$

For the same power *P*,

$$\frac{R_{\text{red}}}{R_{\text{blue}}} = \left(\frac{P\lambda_{\text{red}}}{hc}\right)\left(\frac{hc}{P\lambda_{\text{blue}}}\right) = \frac{\lambda_{\text{red}}}{\lambda_{\text{blue}}} = \frac{650\ \text{nm}}{450\ \text{nm}} = 1.44$$

38.12. Solve: The laser light delivers 2.50×10^{17} photons per second and $100\times10^{-3}\ \text{J}$ of energy per second. Thus, the energy of each photon is

$$\frac{100\times10^{-3}\ \text{J/s}}{2.50\times10^{17}\ \text{s}^{-1}} = 4.00\times10^{-19}\ \text{J}$$

From Equation 38.4, the wavelength of the photons is

$$\lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J s})(3.00\times10^{8}\ \text{m/s})}{4.00\times10^{-19}\ \text{J}} = 4.97\times10^{-7}\ \text{m} = 497\ \text{nm}$$

Assess: The wavelength is in the visible region.

38.13. **Model:** All the light emitted by the light bulb is assumed to have a wavelength of 600 nm.
Solve: The rate of photon emission is

$$R_{\text{photon}} = \frac{\text{P}}{hf} = \frac{5\ \text{W}}{hf} = \frac{5\ \text{W}}{(6.63\times10^{-34}\ \text{J s})(5.0\times10^{14}\ \text{Hz})} = 1.5\times10^{19}\ \text{photons/s} \approx 1\times10^{19}\ \text{photons/s}$$

38.14. **Model:** Use the photon model of *x*-rays. "Directly back toward the source" means $\theta = 180° \Rightarrow \cos\theta = -1$.
Visualize: We need wavelengths for the Compton scattering formula, so we'll convert the initial energy to a wavelength and then convert back from wavelength to energy at the end.
Solve: The photon energy is $E_{\text{photon}} = hf$, and the wavelength and frequency are related by $f = c/\lambda$. Thus the incident wavelength is

$$\lambda_{\text{i}} = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{(75\ \text{keV})(1.6\times10^{-19}\ \text{J/eV})} = 0.01675\ \text{nm}$$

Use the Compton scattering formula.

$$\Delta\lambda = \lambda_{\text{s}} - \lambda_{\text{i}} = \frac{h}{mc}(1-\cos\theta) = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(3.0\times10^{8}\ \text{m/s})}(1-(-1)) = 0.004852\ \text{nm}$$

So the scattered wavelength is

$$\lambda_{\text{s}} = \lambda_{\text{i}} + \Delta\lambda = 0.01675\ \text{nm} + 0.004852\ \text{nm} = 0.02143\ \text{nm}$$

Convert this *x*-ray wavelength back to an energy.

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{0.02143\ \text{nm}} = 9.281\times10^{-15}\ \text{J} = 58\ \text{keV}$$

Assess: We knew the scattered energy would be less than the incident energy, and this answer is in the right range.

38.15. **Model:** Use the photon model of *x*-rays.
Visualize: We need wavelengths for the Compton scattering formula, so we'll convert both energies to wavelengths and then solve for θ.
Solve: The photon energy is $E_{\text{photon}} = hf$, and the wavelength and frequency are related by $f = c/\lambda$. Thus the incident wavelength is

$$\lambda_{\text{i}} = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{(55\ \text{keV})(1.6\times10^{-19}\ \text{J/eV})} = 0.02260\ \text{nm}$$

The scattered wavelength is

$$\lambda_{\text{s}} = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{(50\ \text{keV})(1.6\times10^{-19}\ \text{J/eV})} = 0.02486\ \text{nm}$$

The difference in wavelength is

$$\Delta\lambda = 0.02486\ \text{nm} - 0.02260\ \text{nm} = 0.002260\ \text{nm}$$

Use the Compton scattering formula and solve for θ.

$$\Delta\lambda = \lambda_{\text{s}} - \lambda_{\text{i}} = \frac{h}{mc}(1-\cos\theta) \Rightarrow 1-\cos\theta = \frac{mc\Delta\lambda}{h} \Rightarrow$$

$$\theta = \cos^{-1}\left(1-\frac{mc\Delta\lambda}{h}\right) = \cos^{-1}\left(1-\frac{(9.11\times10^{-31}\ \text{kg})(3.0\times10^{8}\ \text{m/s})(0.002260\ \text{nm})}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}\right) = 86°$$

Assess: This scattering angle is about right for a 10% increase in wavelength.

Section 38.4 Matter Waves and Energy Quantization

38.16. Solve: (a) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J s}}{(9.11\times10^{-31}\ \text{kg})v} = 1.0\times10^{-12}\ \text{m} \Rightarrow v = 7.3\times10^{8}\ \text{m/s}$$

This speed is larger than *c*, indicating a breakdown of de Broglie's equation. This is an acceptable answer if you haven't studied relativity. However, a better approach would be to use the relativistic form for the momentum, $p = \gamma mv$. Hence,

$$\lambda = \frac{h}{\gamma mv} = \frac{6.63\times10^{-34}\ \text{J s}\sqrt{1-v^2/c^2}}{(9.11\times10^{-31}\ \text{kg})v} = 1.0\times10^{-12}\ \text{m}$$

$$\Rightarrow \sqrt{1-v^2/c^2} = \left(\frac{v}{c}\right)\frac{(3.0\times10^{8}\ \text{m/s})(1.0\times10^{-12}\ \text{m})(9.11\times10^{-31}\ \text{kg})}{6.63\times10^{-34}\ \text{J s}} = 0.4122\left(\frac{v}{c}\right)$$

$$\Rightarrow 1-\frac{v^2}{c^2} = 0.170\frac{v^2}{c^2} \Rightarrow v = 0.925c = 2.8\times10^{8}\ \text{m/s}$$

(b) For $\lambda = 1.0\times10^{-9}$ m,

$$\lambda = 1.0\times10^{-9}\ \text{m} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J s}}{(9.11\times10^{-31}\ \text{kg})v} \Rightarrow v = 7.3\times10^{5}\ \text{m/s}$$

(c) Likewise, for $\lambda = 1.0\times10^{-6}$ m, $v = 7.3\times10^{2}$ m/s.

(d) For $\lambda = 1.0\times10^{-3}$ m, $v = 0.73$ m/s.

38.17. Model: Assume the neutron is in free fall.

Solve: The speed of an object that has free fallen 1.0 m from rest is given from the kinematic equation as

$$v_y = \sqrt{2g\Delta y} = \sqrt{2(9.8\ \text{m/s}^2)(1.0\ \text{m})} = 4.427\ \text{m/s}$$

At that speed a neutron has a wavelength of

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J s}}{(1.67\times10^{-27}\ \text{kg})(4.427\ \text{m/s})} = 90\ \text{nm}$$

Assess: The wavelength of the neutron changes as it falls and its speed changes.

38.18. Solve: The de Broglie wavelength is $\lambda = h/mv$. Thus,

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{J s}}{(9.11\times10^{-31}\ \text{kg})(500\times10^{-9}\ \text{m})} = 1456\ \text{m/s}$$

A potential difference of ΔV will raise the kinetic energy of a rest electron by $\frac{1}{2}mv^2$. Thus,

$$e\Delta V = \frac{1}{2}mv^2 \Rightarrow \Delta V = \frac{mv^2}{2e} = \frac{(9.11\times10^{-31}\ \text{kg})(1456\ \text{m/s})^2}{2(1.6\times10^{-19}\ \text{C})} = 6.0\times10^{-6}\ \text{V}$$

Assess: A mere 6.0×10^{-6} V is able to increase an electron's speed to 1456 m/s.

38.19. Solve: (a) The baseball's momentum is $p = mv = (0.200\ \text{kg})(30\ \text{m/s}) = 6.0\ \text{kg m/s}$. The baseball's de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\ \text{J s}}{6.0\ \text{kg m/s}} = 1.1\times10^{-34}\ \text{m}$$

(b) Using $\lambda = h/p = h/mv,$ we have

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{J s}}{(0.200\ \text{kg})(0.20\times10^{-9}\ \text{m})} = 1.7\times10^{-23}\ \text{m/s}$$

38.20. Visualize: Equation 38.16 gives the de Broglie wavelength of a particle in a box.

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, 4, \ldots$$

We are given $\lambda_n = 1.0$ nm and $L = 3.0$ nm.

Solve: Solve the above equation for n.

$$n = \frac{2L}{\lambda_n} = \frac{2(3.0\ \text{nm})}{1.0\ \text{nm}} = 6$$

Assess: 6 is an unsurprising value for a quantum number.

38.21. Solve: The de Broglie wavelength is $\lambda = h/mv.$ Thus

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{J s}}{(1.67\times10^{-27}\ \text{kg})(10\times10^{-15}\ \text{m})} = 3.970\times10^{7}\ \text{m/s}$$

So, the kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\ \text{kg})(3.970\times10^{7}\ \text{m/s})^2 \times \frac{1\ \text{eV}}{1.6\times10^{-19}\ \text{J}} = 8.2\ \text{MeV}$$

38.22. Model: The energy of a confined particle in a one-dimensional box is quantized. Model a nucleus as a one-dimensional box of length $L = 10\ \text{fm} = 1.0\times10^{-14}$ m.

Solve: Protons and neutrons are particles of mass *m* confined in a box. From Equation 38.18, the allowed energies of the protons are

$$E_n = n^2\frac{h^2}{8mL^2} = n^2\frac{(6.63\times10^{-34}\ \text{J s})^2}{8(1.67\times10^{-27}\ \text{kg})(1.0\times10^{-14}\ \text{m})^2} = n^2(3.29\times10^{-13}\ \text{J}) = n^2(2.06\ \text{MeV})$$

The first three energy levels are to two significant figures $E_1 = 2.1$ MeV, $E_2 = 4E_1 = 8.2$ MeV, and $E_3 = 9E_1 = 19$ MeV.

38.23. Model: For a "particle in a box," the energy is quantized.

Solve: The energy of the $n = 1$ state is

$$E_1 = (1)^2\frac{h^2}{8mL^2} = E_{\text{photon}} = \frac{hc}{\lambda} \Rightarrow L = \sqrt{\frac{h\lambda}{8mc}} = \sqrt{\frac{(6.63\times10^{-34}\ \text{J s})(600\times10^{-9}\ \text{m})}{8(9.11\times10^{-31}\ \text{kg})(3.0\times10^{8}\ \text{m/s})}} = 0.427\ \text{nm}$$

Section 38.5 Bohr's Model of Atomic Quantization

38.24. Model: To conserve energy, the emission and the absorption photons must have exactly the energy lost or gained by the atom in the appropriate quantum jumps.

Visualize: The energy of a light quantum is $E = hf = hc/\lambda.$

Solve: (a) The wavelength of the emission photon from the $n = 2$ to $n = 1$ transition is

$$\lambda = \frac{hc}{E_2 - E_1} = \frac{(4.14\times10^{-15}\ \text{eV s})(3.0\times10^{8}\ \text{m/s})}{1.5\ \text{eV}} = 828\ \text{nm}$$

Likewise, $\lambda = 497$ nm for the $3 \rightarrow 2$ transition with $\Delta E = 2.5$ eV, and $\lambda = 311$ nm for the $3 \rightarrow 1$ transition with $\Delta E = 4.0$ eV.

(b) Because the atom in the ground state is in the $n=1$ state, the absorption lines correspond to the $1 \rightarrow 2$ and $1 \rightarrow 3$ transitions. The absorption wavelengths are 828 nm and 311 nm. The $2 \rightarrow 3$ transition is not seen in absorption.

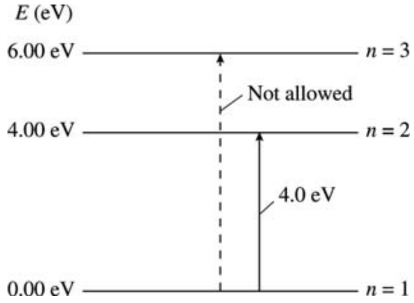
38.25. Model: To conserve energy, the absorption spectrum must have exactly the energy gained by the atom in the quantum jumps.

Solve: (a) An electron with a kinetic energy of 2.00 eV can collide with an atom in the $n=1$ state and raise its energy to the $n=2$ state. This is possible because $E_2 - E_1 = 1.50\text{ eV}$ is less than 2.00 eV. On the other hand, the atom cannot be excited to the $n=3$ state.

(b) The atom will absorb 1.50 eV of energy from the incoming electron, leaving the electron with 0.50 eV of kinetic energy.

38.26. Model: The electron must have $k \geq \Delta E_{\text{atom}}$ to cause collisional excitation. The atom is initially in the $n=1$ ground state.

Visualize:



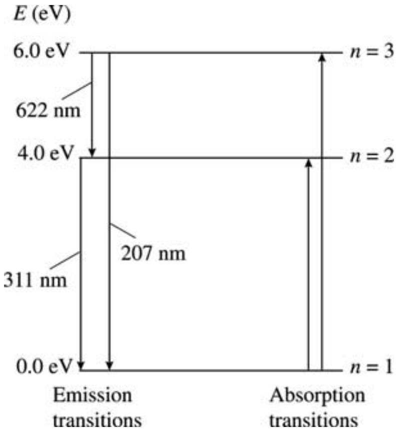
Solve: The kinetic energy of the incoming electron is

$$E = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(9.11\times10^{-31}\text{ kg})(1.30\times10^{6}\text{ m/s})^2 = 7.698\times10^{-19}\text{ J} = 4.81\text{ eV}$$

The electron has enough energy to excite the atom to the $n=2$ stationary state $(E_2 - E_1 = 4.00\text{ eV})$. However, it does not have enough energy to excite the atom into the $n=3$ state, which requires a total energy of 6.00 eV.

38.27. Model: To conserve energy, the emission and the absorption spectra must have exactly the energy lost or gained by the atom in the appropriate quantum jumps.

Solve: (a)



(b) From Equation 38.4, the energy of a light quantum is $E = hf = hc/\lambda.$ We can use this equation to find the emission and absorption wavelengths. The emission energies from the above energy-level diagram are $E_{2\to1} = 4.00$ eV, $E_{3\to1} = 6.00$ eV, and $E_{3\to2} = 2.00$ eV. The wavelength corresponding to the $2 \to 1$ transition is

$$\lambda_{2\to1} = \frac{hc}{E_{21}} = \frac{(4.14\times10^{-15}\text{ eV s})(3.0\times10^{8}\text{ m/s})}{4.00\text{ eV}} = 311\text{ nm}$$

Likewise, $\lambda_{3\to1} = hc/E_{3\to1} = 207$ nm, and $\lambda_{3\to2} = 622$ nm.

(c) Absorption transitions start from the $n=1$ ground state. The energies in the atom's absorption spectrum are $E_{1\to2} = 4.00$ eV and $E_{1\to3} = 6.00$ eV. The corresponding wavelengths are $\lambda_{1\to2} = hc/E_{1\to2} = 311$ nm and $\lambda_{1\to3} = hc/E_{1\to3} = 207$ nm.

Section 38.6 The Bohr Hydrogen Atom

38.28. Model: Assume the electron is in one of the stationary states.

Visualize: The binding energy (0.378 eV) is $|E_n|$; the energies are negative, so $E_n = -0.378$ eV.

Solve: We solve for *n* in Equation 38.33:

$$n^2 = -\frac{E_1}{E_n} = \frac{-13.60\text{ eV}}{-0.378\text{ eV}} = 36 \Rightarrow n = 6$$

Now use Equation 38.26.

$$r_n = n^2 a_\text{B} \Rightarrow r_6 = 6^2(0.0529\text{ nm}) = 1.90\text{ nm}$$

38.29. Model: Assume the electron is in one of the stationary states.

Solve: We solve for *n* in Equation 38.30:

$$n = \frac{v_1}{v_n} = \frac{2.19\times10^{6}\text{ m/s}}{7.3\times10^{5}\text{ m/s}} = 3$$

Look just below Equation 38.29 to find $r_3 = 0.476$ nm.

38.30. Solve: (a) Using the data in Table 38.2, the wavelength of the electron in the $n=1$ state is

$$\lambda_1 = \frac{h}{mv} = \frac{(6.63\times10^{-34}\text{ J s})}{(9.11\times10^{-31}\text{ kg})(2.19\times10^{6}\text{ m/s})} = 0.332\text{ nm}$$

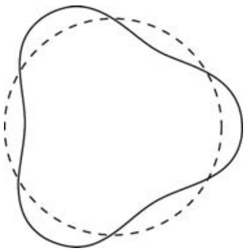
Likewise, $\lambda_2 = 0.665$ nm, and $\lambda_3 = 0.997$ nm.

(b) For $n=1$, the circumference of the orbit is $0.0529\text{ nm}\times 2\pi = 0.332$ nm, which is exactly equal to λ_1. For $n=2$, the circumference of the orbit is

$$0.212\text{ nm}\times2\pi = 1.332\text{ nm} = 2\lambda_2 = 2(0.665\text{ nm})$$

Likewise, the data from part (a) and Table 38.2 shows $3\lambda_3 = 2\pi r_3$.

(c)



38.31. Solve: (a) From Equation 38.29, the radius in state *n* of a hydrogen atom is $r_n = n^2 a_B$. A 100 nm diameter atom has $r_n = 50$ nm. Thus, the quantum state is

$$n = \sqrt{\frac{r_n}{a_B}} = \sqrt{\frac{50 \text{ nm}}{0.0529 \text{ nm}}} = 30.74$$

Since *n* has to be an integer, we obtain $n = 31$.

(b) From Equation 38.31, the electron's speed is

$$v_{31} = \frac{v_1}{31} = \frac{2.19\times10^6 \text{ m/s}}{31} = 7.06\times10^4 \text{ m/s}$$

From Equation 38.34, the electron's energy is

$$E_{31} = -\frac{E_1}{(31)^2} = -\frac{13.60 \text{ eV}}{(31)^2} = -0.0142 \text{ eV}$$

38.32. Solve: From Equation 38.34, the energy of the hydrogen atom in its first excited state is

$$E_2 = \frac{-13.60 \text{ eV}}{(2)^2} = -3.40 \text{ eV}$$

The ionization energy of the hydrogen atom in its first excited state $(n = 2)$ is thus 3.40 eV.

Section 38.7 The Hydrogen Spectrum

38.33. Solve: Photons emitted from the $n = 4$ state start in energy level $n = 4$ and undergo a quantum jump to a lower energy level with $m < 4$. The possibilities are $4 \to 1$, $4 \to 2$, and $4 \to 3$. According to Equation 38.40, the transition $4 \to m$ emits a photon of wavelength.

$$\lambda = \frac{\lambda_0}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)} = \frac{91.18 \text{ nm}}{\left(\frac{1}{m^2} - \frac{1}{16}\right)}$$

These values are given in the table below.

Transition	Wavelength
$4 \to 1$	97.26 nm
$4 \to 2$	486.3 nm
$4 \to 3$	4876 nm

38.34. Solve: For hydrogen-like ions, Equation 38.41 is

$$r_n = n^2 \frac{a_B}{Z} = \frac{(r_n)_H}{Z} \qquad v_n = Z\frac{v_1}{n} = Z \cdot (v_n)_H \qquad E_n = -Z^2\left(\frac{13.60 \text{ eV}}{n^2}\right) = Z^2 (E_n)_H$$

Where $(r_n)_H$, $(v_n)_H$, and $(E_n)_H$ are the values of ordinary hydrogen. He^+ has $Z = 2$. Using Table 38.2 for the values of hydrogen, we get

n	r_n(nm)	v_n(m/s)	E_n(eV)
1	0.026	4.38×10^6	−54.4
2	0.106	2.19×10^6	−13.6
3	0.238	1.46×10^6	−6.0

38.35. Model: Equation 38.40 predicts the absorption spectrum of hydrogen if we let $m = 1$.

$$\lambda_{1\to n} = \frac{91.18 \text{ nm}}{\left(\frac{1}{1^2} - \frac{1}{n^2}\right)} \quad n = 2, 3, 4, \ldots$$

Visualize: The $1 \to 2$ transition will produce the longest wavelength; the $1 \to 3$ transition will produce the second-longest wavelength; and $1 \to 4$ transition will produce the third-longest wavelength.
Solve:

$$\lambda_{1\to 4} = \frac{91.18 \text{ nm}}{\left(\frac{1}{1^2} - \frac{1}{4^2}\right)} = 97.26 \text{ nm}$$

Assess: This result is in the ultraviolet region, as expected for $m = 1$.

Problems

38.36. Visualize: Equation 38.8 gives V_{stop} in terms of f; we replace f with c/λ.

$$V_{\text{stop}} = \frac{hc/\lambda - E_0}{e}$$

The value of E_0 for aluminum is found in Table 38.1: $E_0 = 4.28 \text{ eV} = 6.848 \times 10^{-19}$ J.
Solve: For $\lambda = 250$ nm:

$$V_{\text{stop}} = \frac{hc/\lambda - E_0}{e} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})/(250\times10^{-9} \text{ m}) - 6.848\times10^{-19} \text{ J}}{1.60\times10^{-19} \text{ C}} = 0.693 \text{ V}$$

For $\lambda = 200$ nm:

$$V_{\text{stop}} = \frac{hc/\lambda - E_0}{e} = \frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})(3.00\times10^{8} \text{ m/s})/(200\times10^{-9} \text{ m}) - 6.848\times10^{-19} \text{ J}}{1.60\times10^{-19} \text{ C}} = 1.94 \text{ V}$$

So the change in stopping potential is $1.94 \text{ V} - 0.693 \text{ V} = 1.24 \text{ V}$.
Assess: This answer looks reasonable in light of Figure 38.9.

38.37. Solve: (a) The energy of each photon is

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34} \text{ J s})(3.00\times10^{8} \text{ m/s})}{690\times10^{-9} \text{ m}} = 2.88\times10^{-19} \text{ J}$$

The number of photons in each pulse of light is

$$N = \frac{E_{\text{total}}}{E_{\text{photon}}} = \frac{0.500 \text{ J}}{2.88\times10^{-19} \text{ J}} = 1.7\times10^{18}$$

(b) The *rate* of emission, that is, the number of photons per second is

$$R = \frac{1.7\times10^{18} \text{ photons}}{10\times10^{-9} \text{ s}} = 1.7\times10^{26} \text{ photons/s}$$

38.38. Visualize: Power is the energy per unit time $(P = E/t)$. The total energy comes from a number of photons $(E = NE_{\text{photon}})$ and the energy of each photon is determined by its wavelength $(E = hc/\lambda)$.
Solve: Combine these expressions and solve for the number of photons.

$$P = \frac{E}{t} = \frac{NE_{\text{photon}}}{t} = \frac{N(hc/\lambda)}{t} \Rightarrow N = \frac{P\lambda t}{hc} = \frac{(1.2\times10^{-3} \text{ W})(5.50\times10^{-7} \text{ m})(0.100 \text{ s})}{1.989\times10^{-25} \text{ J}\cdot\text{m}} = 3.3\times10^{14} \text{ photons}$$

Assess: While this seems like a large number, to help keep it in perspective, compare it to the number of molecules in a mole.

38.39. Visualize: Power is related to the total amount of energy and time by $P = E_{\text{total}}/t$. The total amount of energy is related to the energy of a single photon by $E_{\text{total}} = NE_{\text{photon}}$. The energy of a photon is related to the wavelength by $E_{\text{photon}} = hc/\lambda$.

Solve: The power or the flash is

$$P = \frac{E_{\text{total}}}{t} = \frac{NE_{\text{photon}}}{t} = \frac{N}{t}\frac{hc}{\lambda} = \frac{(10^8)}{(0.10\text{ s})}\frac{(19.89\times10^{-26}\text{ J}\cdot\text{m})}{(4.60\times10^{-7}\text{ m})} = 4.3\times10^{-10}\text{ W}$$

Assess: This is a small power rating, but considering that the dinoflagellates might flash numerous times per second, it is a reasonable number.

38.40. Solve: (a) The maximum kinetic energy of photoelectrons is $K_{\text{max}} = hf - E_0$. Substituting the given values,

$$2.8\text{ eV} = \frac{hc}{\lambda} - E_0 \; 1.1\text{ eV} = \frac{hc}{1.5\lambda} - E_0$$

Multiplying the second equation by 1.5 and subtracting the second equation from the first,

$$1.15\text{ eV} = 0.5\,E_0 \Rightarrow E_0 = 2.3\text{ eV}$$

(b) Substituting $E_0 = 2.3\text{ eV}$ into the first equation,

$$2.8\text{ eV} = \frac{(4.14\times10^{-15}\text{ eV s})(3.0\times10^8\text{ m/s})}{\lambda} - 2.3\text{ eV} \Rightarrow \lambda = 244\text{ nm}$$

38.41. Solve: (a) The threshold frequency is $f_0 = E_0/h$. The threshold frequency for potassium and gold are given in the table in part (b).

(b) The corresponding threshold wavelength is $\lambda_0 = c/f_0$. The results of the calculations are in the table below.

Metal	E_0(eV)	f_0(Hz)	λ_0(nm)
Potassium	2.30	5.56×10^{14}	540
Gold	5.10	1.23×10^{15}	244

(c) When light of wavelength λ is incident on the metal, the maximum kinetic energy of the photoelectrons is

$$K_{\text{max}} = \tfrac{1}{2}mv_{\text{max}}^2 = hf - E_0 = \frac{hc}{\lambda} - E_0 \Rightarrow v_{\text{max}} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda} - E_0\right)}$$

E_0 must be converted to SI units of joules before this formula can be used. v_{max} for potassium and gold are given in the table in part (d).

(d) The stopping potential is

$$V_{\text{stop}} = \frac{K_{\text{max}}}{e} = \frac{1}{e}\left(\frac{hc}{\lambda} - E_0\right)$$

where again E_0 has to be joules. The results of the calculations are in the table below.

Metal	E_0(J)	v_{max}(m/s)	V_{stop}(V)
Potassium	3.68×10^{-19}	10.8×10^5	3.35
Gold	8.16×10^{-19}	4.4×10^5	0.55

38.42. Solve: (a) The stopping potential is

$$V_{\text{stop}} = \frac{h}{e} f - \frac{h}{e} f_0$$

A graph of V_{stop} versus frequency *f* should be linear with *x*-intercept f_0 and slope h/e. Since the *x*-intercept is $f_0 = 1.0\times10^{15}$ Hz, the work function is

$$E_0 = hf_0 = (4.14\times10^{-15}\text{ eV s})(1.0\times10^{15}\text{ Hz}) = 4.14\text{ eV}$$

(b) The slope of the graph is

$$\frac{\Delta V_{\text{stop}}}{\Delta f} = \frac{8.0\text{ V} - 0\text{ V}}{3.0\times10^{15}\text{ Hz} - 1.0\times10^{15}\text{ Hz}} = 4.0\times10^{-15}\text{ V s}$$

Because the slope of the V_{stop} versus *f* graph is h/e, an experimental value of Planck's constant is

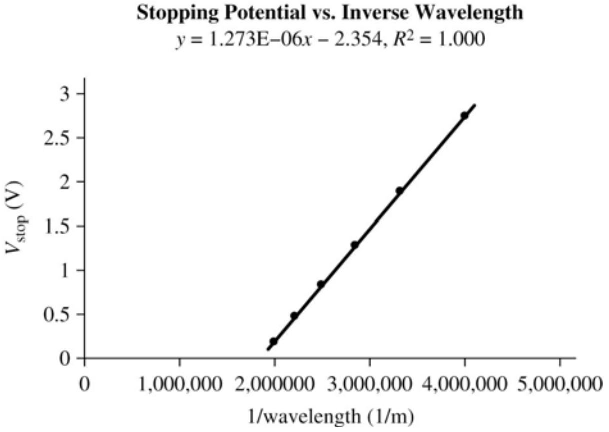
$$h = e(4.0\times10^{-15}\text{ V s}) = (1.6\times10^{-19}\text{ C})(4.0\times10^{-15}\text{ V s}) = 6.4\times10^{-34}\text{ J s}$$

Assess: This value of the Planck's constant is about 3.5% lower than the accepted value.

38.43. Solve: The photoelectric stopping potential is

$$V_{\text{stop}} = \frac{hf - E_0}{e} = \frac{hc}{e}\frac{1}{\lambda} - \frac{E_0}{e}$$

where we used $f = c/\lambda$. Thus a plot of V_{stop} versus $1/\lambda$ should be linear with a slope of hc/e and a *y*-intercept of E_0/e.



(a) The linear fit is seen to be quite good. From the experimental *y*-intercept of -2.35 V, the cathode's work function is

$$E_0 = e(2.35\text{ V}) = 2.35\text{ eV}$$

With a slight bit of experimental error, the cathode appears to be made of potassium $(E_0 = 2.30\text{ eV})$.

(b) The slope is seen to be 1.273×10^{-6} V m, from which

$$h = \frac{e\cdot\text{slope}}{c} = \frac{(1.60\times10^{-19}\text{ C})(1.273\times10^{-6}\text{ V m})}{3.00\times10^{8}\text{ m/s}} = 6.79\times10^{-34}\text{ J s}$$

Assess: The value of *h* is a couple percent higher than the accepted value.

38.44. Model: Use the photon model of radio waves.
Visualize: The ratio of power absorbed by the antenna to the power emitted by the transmitter is equal to the ratio of the cross sectional area of the antenna (vertically) to the area of a sphere around the transmitter with radius *d*. The cross section area of the antenna is the height multiplied by the width (diameter): $A_{\text{ant}} = 1.5\text{ m}\times3.0\text{ mm} = 0.0045\text{ m}^2$. We are given $d = 15$ km.

Solve: The photon energy is $E_{\text{photon}} = hf = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(550\ \text{kHz}) = 3.646\times10^{-28}\ \text{J}$. Now compute the number of photons hitting the antenna per second.

$$\frac{\#\ \text{photons}}{\Delta t} = \frac{\text{power absorbed}}{\text{energy per photon}} = \frac{P_{\text{abs}}}{E/\text{photon}} = \frac{P_{\text{emit}}(A_{\text{ant}}/A_{\text{emit}})}{E/\text{photon}} = \frac{P_{\text{emit}}}{E/\text{photon}}\frac{A_{\text{ant}}}{A_{\text{emit}}}$$

$$= \frac{75\ \text{kW}}{3.646\times10^{-28}\ \text{J}}\ \frac{0.0045\ \text{m}^2}{4\pi(15\ \text{km})^2} = 3.3\times10^{20}\ \text{photons/s}$$

Assess: This scattering angle is about right for a 10% increase in wavelength.

38.45. Model: Use the photon model of radio waves.
Visualize: We are given $f = 160$ GHz.

Solve: The photon energy is $E_{\text{photon}} = hf = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(160\ \text{GHz}) = 1.061\times10^{-22}\ \text{J}$.

$$E = (\text{energy per photon})(\#\ \text{photons per volume})(\text{volume})$$
$$= (1.061\times10^{-22}\ \text{J/photon})\left(\frac{450\ \text{photons}}{1\ \text{cm}^3}\right)(95\ \text{m}^2\times2.5\ \text{m}) = 1.13\times10^{-11}\ \text{J} = 71\ \text{MeV}$$

Assess: This is not a lot of energy on one room, but it is a lot over the whole universe. You can see some of this cosmic microwave background on an old-fashioned rabbit ears antenna TV; it is part of the snow you see when the TV is not tuned to a broadcast channel.

38.46. Model: Use the photon model of *x* gamma rays. "Back toward the source" means $\theta = 180° \Rightarrow \cos\theta = -1$.
Visualize: We need wavelengths for the Compton scattering formula, so we'll convert the initial energy to a wavelength and then convert back from wavelength to energy at the end. The difference in energy between the incident gamma ray and the scattered gamma ray is given to the electron (which started at rest).
Solve: The photon energy is $E_{\text{photon}} = hf$, and the wavelength and frequency are related by $f = c/\lambda$. Thus the incident wavelength is

$$\lambda_{\text{i}} = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^8\ \text{m/s})}{(350\ \text{keV})(1.6\times10^{-19}\ \text{J/eV})} = 3.552\ \text{pm}$$

Use the Compton scattering formula.

$$\Delta\lambda = \lambda_{\text{s}} - \lambda_{\text{i}} = \frac{h}{mc}(1-\cos\theta) = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(3.0\times10^8\ \text{m/s})}(1-(-1)) = 4.852\ \text{pm}$$

So the scattered wavelength is

$$\lambda_{\text{s}} = \lambda_{\text{i}} + \Delta\lambda = 3.551\ \text{pm} + 4.852\ \text{pm} = 8.4038\ \text{pm}$$

Convert this gamma ray wavelength back to an energy of the scattered photon.

$$E_{\text{photon}} = \frac{hc}{\lambda_{\text{s}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^8\ \text{m/s})}{8.4038\ \text{pm}} = 2.3668\times10^{-14}\ \text{J} = 147.9\ \text{keV}$$

The difference in energy of the photons becomes the kinetic energy of the electron.

$$\Delta E_{\text{photon}} = 350\ \text{keV} - 147.9\ \text{keV} = 202.1\ \text{keV} = 3.233\times10^{-14}\ \text{J} = K_{\text{electron}}$$

Now we solve for *v* using the relativistic formula for kinetic energy of the electron.

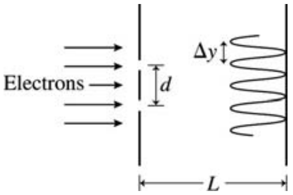
$$K = (\gamma_{\text{p}} - 1)mc^2 \Rightarrow \frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow 1 - \beta^2 + \left(\frac{K}{mc^2} + 1\right)^{-2} \Rightarrow$$

$$\beta = \sqrt{1 - \left(\frac{K}{mc^2} - 1\right)^{-2}} = \sqrt{1 - \left(\frac{3.233\times10^{-14}\ \text{J}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(3.0\times10^8\ \text{m/s}\right)^2} - 1\right)^{-2}} = 0.6969 \Rightarrow$$

$$v = 0.6969c = (0.6969)(3.0\times10^8\ \text{m/s}) = 2.1\times10^8\ \text{m/s}$$

Assess: The answer is in the expected range for a 200 keV electron.

38.47. Model: Electrons have both particle-like and wave-like properties.
Visualize:



Please refer to Figure 38.12.

Solve: (a) The kinetic energy is $K = \frac{1}{2}mv^2 = 50\ \text{keV} = 8.0\times10^{-15}\ \text{J}$. Using this formula, the electron's speed is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(8.0\times10^{-15}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = 1.32\times10^8\ \text{m/s} \approx 1.3\times10^8\ \text{m/s}$$

(b) From Equation 33.7, the fringe spacing in a double-slit interference experiment is $\Delta y = \lambda L/d$, where d is the slit separation and L is the distance to the viewing screen. The wavelength of the electrons is their de Broglie wavelength. We have

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J s}}{(9.11\times10^{-31}\ \text{kg})(1.32\times10^8\ \text{m/s})} = 5.5\times10^{-12}\ \text{m}$$

$$\Rightarrow \Delta y = \frac{\lambda L}{d} = \frac{(5.5\times10^{-12}\ \text{m})(1.0\ \text{m})}{1.0\times10^{-6}\ \text{m}} = 5.5\times10^{-6}\ \text{m} = 5.5\ \mu\text{m}$$

38.48. Model: The electrons are wave-like and diffract through a single slit. Assume the electrons are not relativistic.
Visualize: We are given $L = 0.50$ m, $a = 15$ nm, and $w = 3.3$ mm.

Solve: From the earlier chapter on diffraction we find the width of the central maximum for diffraction through a single slit. Solve for λ.

$$w = \frac{2\lambda L}{a} \Rightarrow \lambda = \frac{aw}{2L} = \frac{(15\ \text{nm})(3.3\ \text{mm})}{2(0.50\ \text{m})} = 0.0495\ \text{nm}$$

This wavelength is the de Broglie wavelength $\lambda = h/p$. Solve this for p.

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.0495\ \text{nm}} = 1.339\times10^{-23}\ \text{kg}\cdot\text{m/s}$$

Use this value of p to find the kinetic energy.

$$K = \frac{p^2}{2m} = \frac{(1.339\times10^{-23}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 9.85\times10^{-17}\ \text{J} = 620\ \text{eV}$$

Assess: These are not very energetic electrons, so they are not relativistic.

38.49. Model: Use the wave model for neutrons; they will interfere as they travel through the two-slit apparatus.
Visualize: From the prior chapter on the wave model of light we know that the fringe spacing in a two-slit experiment with slit spacing d and screen distance L is $\Delta y = \dfrac{\lambda L}{d}$. Using the scale on the figure, the five tallest peaks, which are $4\Delta y$, appear to span about $250\ \mu\text{m}$, so $\Delta y = 250\ \mu\text{m} = 4 \approx 63\ \mu\text{m}$. We are given $d = 0.10$ mm and $L = 3.5$ m.

Solve: Solve the de Broglie wavelength $\lambda = \dfrac{h}{mv}$ for v.

$$v = \frac{h}{m\lambda} = \frac{h}{m\left(\dfrac{d}{L}\Delta y\right)} = \frac{6.63\times10^{-34}\text{ J s}}{(1.67\times10^{-27}\text{ kg})\left[\dfrac{0.10\text{ mm}}{(3.5\text{ m})}(63\,\mu\text{m})\right]} = 220\text{ m/s} \approx 200\text{ m/s}$$

Because of the estimation in reading the fringe spacing from the figure, we round the answer to one significant figure.
Assess: This speed seems reasonable for neutrons.

38.50. Model: The energy of a confined particle in a one-dimensional box is quantized.
Solve: The energy of the nth quantum state of a particle in a box is $E_n = n^2E_1$, where E_1 is the lowest energy level. The energies 12 eV, 27 eV, and 48 eV have the ratios 4:9:16. Thus, they are the $n = 2$, $n = 3$, and $n = 4$ states of an electron that has $E_1 = 3\text{ eV} = 4.8\times10^{-19}\text{ J}$. The lowest energy level is

$$E_1 = \frac{h^2}{8mL^2} \Rightarrow L = \sqrt{\frac{h^2}{8mE_1}} = \sqrt{\frac{(6.63\times10^{-34}\text{ J s})^2}{8(9.11\times10^{-31}\text{ kg})(4.8\times10^{-19}\text{ J})}} = 3.5\times10^{-10}\text{ m} = 0.35\text{ nm}$$

38.51. Model: The muons have quantized energy.
Visualize: We are given $L = 15\text{ pm}$ and $m_\mu = 207m_\text{e}$.
Solve: The energy of a particle in a one-dimensional box is

$$E_n = n^2\frac{h^2}{8mL^2} \Rightarrow$$
$$E_2 - E_1 = \left(\frac{h^2}{8mL^2}\right)(2^2 - 1^2) = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{8(207)(9.11\times10^{-31}\text{ kg})(15\text{ pm})^2}(3) = 3.885\times10^{-18}\text{ J}$$

This energy difference becomes the energy of the emitted photon.

$$E = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.0\times10^8\text{ m/s})}{3.885\times10^{-18}\text{ J}} = 51\text{ nm}$$

Assess: This wavelength is too short to be in the visible range. Muons decay with a short half-life; this kind of decay helped experimentally verify special relativity.

38.52. Model: The energy of the emitted gamma-ray photon is exactly equal to the energy between levels 1 and 2.
Solve: From Equation 38.18, the energy levels of the proton are

$$E_n = n^2\frac{h^2}{8\,mL^2} \quad n = 1, 2, 3, \ldots$$

The energy of the emitted photon is

$$E_2 - E_1 = \frac{4h^2}{8mL^2} - \frac{h^2}{8mL^2} = \frac{3h^2}{8mL^2}$$
$$\Rightarrow L = \sqrt{\frac{3h^2}{8m(E_2 - E_1)}} = \sqrt{\frac{3(6.63\times10^{-34}\text{ J s})^2}{8(1.67\times10^{-27}\text{ kg})(2.0\times10^6\text{ eV})}\times\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}} = 1.8\times10^{-14}\text{ m} = 18\text{ fm}$$

Assess: This is roughly the size of a typical nucleus.

38.53. Model: The energy of the emitted photon is exactly equal to the energy between the energy levels 1 and 2.
Solve: From Equation 38.18, the energy levels of the electron are

$$E_n = n^2\frac{h^2}{8\,mL^2} \quad n = 1, 2, 3, \ldots$$

The energy of the emitted photon is

$$E_2 - E_1 = hf = h\frac{c}{\lambda} = \frac{4h^2}{8mL^2} - \frac{h^2}{8mL^2} = \frac{3h^2}{8mL^2}$$

$$\Rightarrow L = \sqrt{\frac{3h^2}{8m}\left(\frac{\lambda}{hc}\right)} = \sqrt{\frac{3h\lambda}{8mc}} = \sqrt{\frac{3(6.63\times10^{-34}\text{ J s})(200\times10^{-9}\text{ m})}{8(9.11\times10^{-31}\text{ kg})(3.0\times10^{8}\text{ m/s})}} = 4.27\times10^{-10}\text{ m} = 0.427\text{ nm}$$

38.54. Visualize: The allowed energies of a particle in a box are

$$E_n = \frac{h^2}{8mL^2}n^2$$

We are given $L = 10$ nm and $n = 1$. We look up the mass of a sodium ion: $m_{\text{Na}} = 3.82\times10^{-26}$ kg.

Solve:

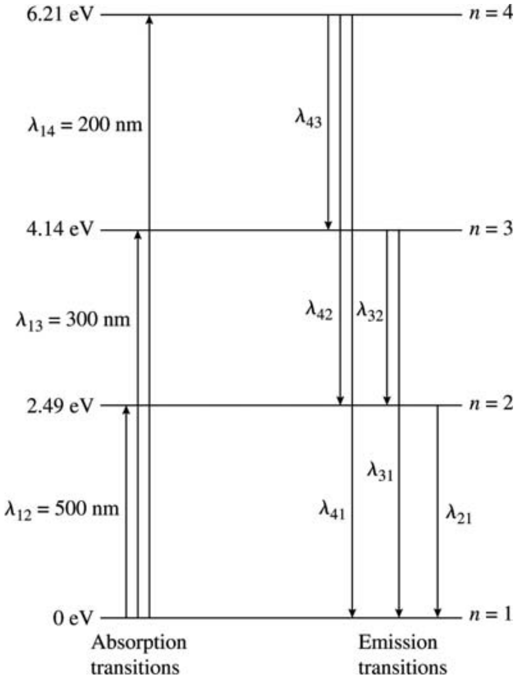
$$E_1 = \frac{h^2}{8mL^2}(1)^2 = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{8(3.82\times10^{-26}\text{ kg})(10\text{ nm})^2} = 1.4\times10^{-26}\text{ J} = 9.0\times10^{-8}\text{ eV}$$

This energy is much smaller than the 70×10^{-3} eV, which is the potential energy of a sodium ion due to the membrane potential.

Assess: The sodium ion does not have to be in the ground state and could have n^2 times as much kinetic energy.

38.55. Model: Photons are emitted when an atom undergoes a quantum jump from a higher energy level to a lower energy level. On the other hand, photons are absorbed in a quantum jump from a lower energy level to a higher energy level. Because most of the atoms are in the $n = 1$ ground state, the only quantum jumps in the absorption spectrum start from the $n = 1$ state.

Solve: (a) Using $E_{\text{photon}} = \frac{hc}{\lambda} = \Delta E_{\text{atom}}$, the 3 wavelengths in the absorption spectrum give 2.49 eV, 4.14 eV, and 6.21 eV as the energies of the $n = 2$, 3, and 4 energy levels.



(b) The emission spectrum of the atom will contain the following wavelengths:

$$\lambda_{41} = \frac{hc}{E_4 - E_1} = \frac{hc}{6.21\text{ eV}} = 200\text{ nm} \qquad \lambda_{31} = \frac{hc}{E_3 - E_1} = \frac{hc}{4.14\text{ eV}} = 300\text{ nm}$$

$$\lambda_{21} = \frac{hc}{E_2 - E_1} = \frac{hc}{2.49\text{ eV}} = 500\text{ nm} \qquad \lambda_{42} = \frac{hc}{E_4 - E_2} = \frac{hc}{3.72\text{ eV}} = 334\text{ nm}$$

$$\lambda_{43} = \frac{hc}{E_4 - E_3} = \frac{hc}{2.07\text{ eV}} = 601\text{ nm} \qquad \lambda_{32} = \frac{hc}{E_3 - E_2} = \frac{hc}{1.65\text{ eV}} = 753\text{ nm}$$

38.56. Model: Photons are emitted when an atom undergoes a quantum jump from a higher energy level to a lower energy level. On the other hand, photons are absorbed in a quantum jump from a lower energy level to a higher energy level. Because most of the atoms are in the $n = 1$ ground state, the only quantum jumps in the absorption spectrum start from the $n = 1$ state.

Solve: (a) The ionization energy is $|E_1| = 6.5\text{ eV}$.

(b) The absorption spectrum consists of the transitions $1 \to 2$ and $1 \to 3$ from the ground state to excited states. According to the Bohr model, the required photon frequency and wavelength are

$$f = \frac{\Delta E}{h} \Rightarrow \lambda = \frac{c}{f} = \frac{hc}{\Delta E}$$

where $\Delta E = E_\text{f} - E_\text{i}$ is the energy change of the atom. Using the energies given in the figure, we calculated the values in the table below.

Transition	E_f (eV)	E_i (eV)	ΔE (eV)	λ (nm)
$1 \to 2$	–3.0	–6.5	3.5	355
$1 \to 3$	–2.0	–6.5	4.5	276

(c) Both wavelengths are ultraviolet $(\lambda < 400\text{ nm})$.

38.57. Solve: A photon with wavelength $\lambda = 1240\text{ nm}$ has an energy $E_\text{photon} = hf = hc/\lambda = 1.0\text{ eV}$. Because E_photon must exactly match ΔE of the atom, a 1240 nm photon can be emitted only in a $3 \to 2$ transition. So, after the collision the atom was in the $n = 3$ state. Before the collision, the atom was in its ground state $(n = 1)$. Thus, an electron with $v_\text{i} = 1.4 \times 10^6\text{ m/s}$ collided with the atom in the $n = 1$ state. The atom gained 4.5 eV in the collision as it was excited from the $n = 1$ to $n = 3$, so the electron lost $4.5\text{ eV} = 7.20 \times 10^{-19}\text{ J}$ of kinetic energy. Initially, the kinetic energy of the electron was

$$K_\text{i} = \tfrac{1}{2} m_\text{elec} v_\text{i}^2 = \tfrac{1}{2}(9.11 \times 10^{-31}\text{ kg})(1.40 \times 10^6\text{ m/s})^2 = 8.93 \times 10^{-19}\text{ J}$$

After losing $7.20 \times 10^{-19}\text{ J}$ in the collision, the kinetic energy is

$$K_\text{f} = K_\text{i} - 7.20 \times 10^{-19}\text{ J} = 1.73 \times 10^{-19}\text{ J} = \tfrac{1}{2} m_\text{elec} v_\text{f}^2 \Rightarrow v_\text{f} = \sqrt{\frac{2K_\text{f}}{m_\text{elec}}} = \sqrt{\frac{2(1.73 \times 10^{-19}\text{ J})}{9.11 \times 10^{-31}\text{ kg}}} = 6.2 \times 10^5\text{ m/s}$$

38.58. Solve: Equation 38.32 is

$$E_n = \frac{1}{2} m \left(\frac{\hbar^2}{m^2 a_\text{B}^2 n^2} \right) - \frac{e^2}{4\pi\varepsilon_0 n^2 a_\text{B}} = \frac{1}{2} \frac{\hbar^2}{m a_\text{B}} \left(\frac{1}{n^2 a_\text{B}} \right) - \frac{e^2}{4\pi\varepsilon_0 n^2 a_\text{B}}$$

The Bohr radius is

$$a_\text{B} = \frac{4\pi\varepsilon_0 \hbar^2}{me^2} \Rightarrow \frac{\hbar^2}{m a_\text{B}} = \frac{e^2}{4\pi\varepsilon_0} \Rightarrow E_n = \frac{1}{2} \left(\frac{e^2}{4\pi\varepsilon_0} \right) \left(\frac{1}{n^2 a_\text{B}} \right) - \left(\frac{e^2}{4\pi\varepsilon_0} \right) \left(\frac{1}{n^2 a_\text{B}} \right) = -\frac{1}{2a_\text{B}} \left(\frac{e^2}{4\pi\varepsilon_0} \right) \frac{1}{n^2} = -\frac{1}{n^2} \left(\frac{1}{4\pi\varepsilon_0} \frac{e^2}{2a_\text{B}} \right)$$

38.59. Solve: The wavelengths in the hydrogen spectrum are given by

$$\lambda_{n\to m} = \frac{91.18\text{ nm}}{1/m^2 - 1/n^2}$$

For each m, the wavelengths range from a maximum value when $n = m+1$ to a minimum value (the series limit) when $n \to \infty$. A few calculations reveal the following behavior:

m	$\lambda_{\max}$ for $n = m+1$	$\lambda_{\min}$ for $n = \infty$
1	122 nm	91 nm
2	656 nm	365 nm
3	1876 nm	822 nm
4	4050 nm	1459 nm

We see that the *only* visible wavelengths (400–700 nm) in the hydrogen spectrum occur for $m = 2$, starting with $n = 3$. We can calculate that $\lambda_{3\to2} = 656.5$ nm, $\lambda_{4\to2} = 486.3$ nm, $\lambda_{5\to2} = 434.2$ nm, and $\lambda_{6\to2} = 410.3$ nm. The next transition, $6 \to 2$, has $\lambda < 400$ nm. Thus these are the only four visible wavelengths in the hydrogen spectrum.

38.60. Model: The speed of the incoming electron is not relativistic, so use $K = \frac{1}{2}mv^2$.

Visualize: The kinetic energy of the incoming electron is $K = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(2.1\times10^{6}\text{ m/s})^2 = 2.01\times10^{-18}\text{ J} = 12.55\text{ eV}$.

Solve: The orbital electron can take most of the energy of the incoming electron and jump up to level $n = 3$ because $n = 3$ is 12.09 eV higher than $n = 1$. The orbital electron can't make it up to $n = 4$ because that would require 12.75 eV.

Now the atom undergoes a quantum jump of $\Delta n = 1$ so it drops from $n = 3$ to $n = 2$. This transition has an energy difference of $-1.51\text{ eV} - (-3.40\text{ eV}) = 1.89\text{ eV}$.

$$\lambda = \frac{hc}{E} = \frac{(4.14\times10^{-15}\text{ eV s})(3.00\times10^{8}\text{ m/s})}{1.89\text{ eV}} = 657\text{ nm}$$

Assess: This wavelength is in the visible region of the spectrum.

38.61. Solve: (a) From Equation 38.40, the wavelengths of the emission spectrum are

$$\lambda_{n\to m} = \frac{91.18\text{ nm}}{m^{-2} - n^{-2}} \quad m = 1, 2, 3, \ldots \; n = m+1, m+2, \ldots$$

For the $200 \to 199$ transition,

$$\lambda_{200\to199} = \frac{91.18\text{ nm}}{(199)^{-2} - (200)^{-2}} = 0.362\text{ m}$$

(b) For the $2 \to 199$ transition,

$$\lambda_{2\to199} = \frac{91.18\text{ nm}}{(2)^{-2} - (199)^{-2}} = 4.000404071\,(91.18\text{ nm})$$

Likewise, $\lambda_{2\to200} = 4.00040004(91.18\text{ nm})$. The difference in the wavelengths of these two transitions is $0.0000041\times(91.18\text{ nm}) = 0.000368\text{ nm}$.

38.62. Solve: (a) The orbital period of an electron in the n state is

$$T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi r_1 n^2}{v_1/n} = \frac{2\pi r_1}{v_1}n^3 = T_1 n^3 = \frac{2\pi(0.0529\text{ nm})}{2.19\times10^{6}\text{ m/s}}n^3 = (1.52\times10^{-16}\text{ s})n^3$$

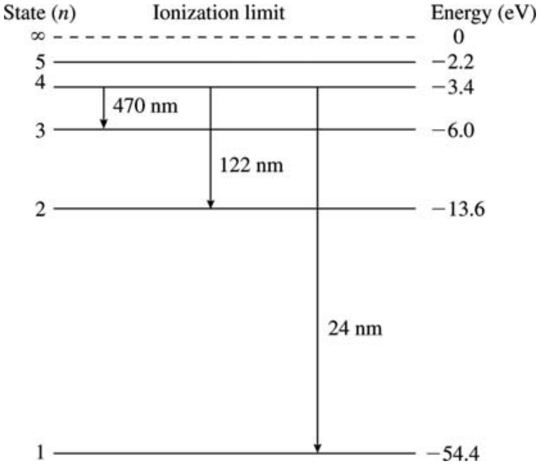
where we obtained the expression for the radius and speed from Equations 38.30 and 38.31. Thus, $T_1 = 1.52\times10^{-16}$ s.

(b) In the $n = 2$ state, the orbital period of the electron is $T_2 = (1.52\times10^{-16}\text{ s})(2)^3 = 1.216\times10^{-15}$ s. The number of revolutions before the electron makes a quantum jump to the $n = 1$ state is

$$\frac{1.6\times10^{-9}\text{ s}}{1.216\times10^{-15}\text{ s}} = 1.32\times10^{6}\text{ revolutions}$$

$0.0000041\times(91.18\text{ nm}) = 0.000368\text{ nm}.$

38.63. Visualize:



Solve: (a) He^+ is a hydrogen-like ion with $Z = 2$. From Equation 38.41, its energy levels are

$$E_n = -Z^2\frac{13.60\text{ eV}}{n^2} = -\frac{54.4\text{ eV}}{n^2}$$

It is straightforward to compute $E_1 = -54.4$ eV, $E_2 = -13.6$ eV, $E_3 = -6.04$ eV, $E_4 = -3.40$ eV, and $E_5 = -2.18$ eV.

(b) The ionization energy is $|E_1| = 54.4$ eV. The ionization limit is shown as $n = \infty$.

(c) The possible transitions from $n = 4$ are $4 \to 3$, $4 \to 2$, and $4 \to 1$.

(d) The wavelengths of these transitions are $\lambda = hc/\Delta E$: The computed values are shown in the table below.

Transition	E_f(eV)	E_i(eV)	ΔE(eV)	λ(nm)
$4 \to 3$	–6.04	–3.40	2.64	470
$4 \to 2$	–13.6	–3.40	10.2	122
$4 \to 1$	–54.4	–3.40	51.0	24

38.64. Model: The Balmer series has 2 as the lower level.
Visualize: For He^+, $Z = 2$. For hydrogen-like ions we are given

$$\lambda_0 = \frac{91.18\text{ nm}}{Z^2} = \frac{91.18\text{ nm}}{2^2} = 22.795\text{ nm}$$

Solve: The wavelength of the emitted photons is

$$\lambda_{n\to m} = \frac{\lambda_0}{\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)}$$

$$\lambda_{3\to2} = \frac{22.795\text{ nm}}{\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right)} = 164\text{ nm} \qquad \lambda_{4\to2} = \frac{22.795\text{ nm}}{\left(\dfrac{1}{2^2} - \dfrac{1}{4^2}\right)} = 122\text{ nm} \qquad \lambda_{5\to2} = \frac{22.795\text{ nm}}{\left(\dfrac{1}{2^2} - \dfrac{1}{5^2}\right)} = 109\text{ nm}$$

Assess: The answer for $3 \to 2$ is the same as given in the table in the book. These answers are all in the ultraviolet region of the spectrum.

38.65. Visualize: A neutral oxygen atom that has lost 7 electrons is a hydrogen-like atom because it has one electron going around a nucleus with $Z = 8$.

Solve: The energy levels for the O^{+7} ion are

$$E_n = -\frac{13.60\ Z^2\ \text{eV}}{n^2} = -\frac{13.60(8)^2\ \text{eV}}{n^2} = -\frac{870.4\ \text{eV}}{n^2}$$

The energy of the $3 \to 2$ transition is

$$E_3 - E_2 = -870.4\ \text{eV}\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = 120.89\ \text{eV} = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{(4.14\times10^{-15}\ \text{eV})(3.0\times10^8\ \text{m/s})}{120.89\ \text{eV}} = 10.28\ \text{nm}$$

Likewise for the $4 \to 2$ transition,

$$E_4 - E_2 = -870.4\ \text{eV}\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 163.2\ \text{eV}$$

The wavelength for this transition $\lambda = 7.62$ nm. For the $5 \to 2$ transition, $E_5 - E_2 = 182.78$ eV and $\lambda = 6.80$ nm. All the wavelengths are in the ultraviolet range.

38.66. Solve: (a) The muon acts like an electron in a hydrogen-like ion of charge Z *except* that the muon's mass is $m_\mu = 207 m_\text{e}$. The different mass changes the Bohr radius to

$$(a_\text{B})_\mu = \frac{4\pi\varepsilon_0\hbar^2}{m_\mu e^2} = \frac{m_e}{m_\mu}\frac{4\pi\varepsilon_0\hbar^2}{m_\text{e}e^2} = \frac{1}{207}a_\text{B} = \frac{1}{207}(0.0529\ \text{nm}) = 2.56\times10^{-4}\ \text{nm}$$

From Equation 38.41, the ground state radius of the muon in the carbon is

$$r_1 = \frac{(a_\text{B})_\mu}{Z} = \frac{2.56\times10^{-4}\ \text{nm}}{6} = 4.26\times10^{-5}\ \text{nm}$$

The speed in the ground state is

$$v_1 = \frac{\hbar}{m_\mu r_1} = \frac{6.63\times10^{-34}\ \text{J s}}{2\pi(207\times9.11\times10^{-31}\ \text{kg})(4.26\times10^{-5}\ \text{nm})} = 1.31\times10^7\ \text{m/s}$$

(b) From Equations 38.34 and 38.41, the energy levels are $E_n = -Z^2(E_1)_\mu/n^2$. Because of the different mass,

$$(E_1)_\mu = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2(a_\text{B})_\mu} = \frac{a_\text{B}}{(a_\text{B})_\mu}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2a_\text{B}} = 207(13.60\ \text{eV}) = 2815\ \text{eV}$$

For $Z = 6$, the $n = 1$ and $n = 2$ energy levels are

$$E_1 = -\frac{(36)2815\ \text{eV}}{1^2} = -101{,}300\ \text{eV} \qquad E_2 = -\frac{(36)2815\ \text{eV}}{2^2} = -25{,}300\ \text{eV}$$

The photon emitted in a $2 \to 1$ transition has an energy $E_\text{photon} = E_2 - E_1 = 76{,}000$ eV. The wavelength is

$$\lambda = \frac{hc}{E_\text{photon}} = \frac{(6.63\times10^{-34}\ \text{J s})(3.0\times10^8\ \text{m/s})}{76{,}000\ \text{eV}} = 1.64\times10^{-11}\ \text{m} = 0.0164\ \text{nm}$$

(c) This is an x-ray photon.

(d) For circular motion, the orbital period is

$$T = \frac{2\pi r_1}{v_1} = \frac{2\pi(4.26\times10^{-5}\ \text{nm})}{1.31\times10^{7}\ \text{m/s}} = 2.04\times10^{-20}\ \text{s}$$

The number of orbits completed during the muon's half-life is

$$N = \frac{\Delta t}{T} = \frac{1.5\times10^{-6}\ \text{s}}{2.04\times10^{-20}\ \text{s}} = 7.3\times10^{13}$$

This is such an incredibly large number of orbits that it "makes sense" to use the Bohr model.

38.67. Model: An inelastic collision between the atoms causes the electrons to be excited.
Solve: From Example 38.12, a wavelength of 121.6 nm corresponds to a $2 \to 1$ transition. Because the atoms were in their ground states before colliding, each atom lost kinetic energy $\Delta K = E_2 - E_1$ and, in the process, excited the electron from $n=1$ to $n=2$, from where it subsequently emitted the photon. Because the atoms each end at rest, $\Delta K = K_\text{i} = \frac{1}{2}mv^2$. Thus

$$\tfrac{1}{2}m_\text{atom}v^2 = E_2 - E_1 = 10.2\ \text{eV}\times\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}} = 1.632\times10^{-18}\ \text{J}$$

$$\Rightarrow\quad v = \sqrt{\frac{2(E_2-E_1)}{m_\text{atom}}} = \sqrt{\frac{2(1.632\times10^{-18}\ \text{J})}{1.67\times10^{-27}\ \text{kg}}} = 44{,}200\ \text{m/s}$$

38.68. Solve: (a) Nearly all atoms spend nearly all of their time in the ground state $(n=1)$. To cause an emission from the $n=3$ state, the electrons must excite hydrogen atoms from the $n=1$ state to the $n=3$ state. The energy gained by each atom is

$$E_3 - E_1 = -1.51\ \text{eV} - (-13.60\ \text{eV}) = 12.09\ \text{eV}$$

This means the electrons must each lose 12.09 eV of kinetic energy. Thus the electrons must have *at least* $K_\text{min} = 12.09\ \text{eV}$ of kinetic energy to cause the emission of 656 nm light. The minimum speed of the electrons is

$$v_\text{min} = \sqrt{\frac{2K_\text{min}}{m_\text{elec}}} = \sqrt{\frac{2(12.09\ \text{eV})}{9.11\times10^{-31}\ \text{kg}}\times\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}} = 2.06\times10^{6}\ \text{m/s}$$

(b) An electron gains 12.09 eV of kinetic energy by being accelerated through a potential difference of 12.09 V. This is simply the definition of *electron volt*.

Challenge Problems

38.69. Model: Use the wave model for electrons as they diffract through the circular aperture.
Visualize: From the prior chapter on the wave model of light we know that in a circular aperture experiment the variables are related by $w = \frac{2.44\lambda L}{D}$, where D is the diameter of the aperture. The energy of an electron accelerated from rest through 250 V is $250\ \text{eV} = 4.00\times10^{-17}\ \text{J}$.
Solve: The speed of the electron is obtained from

$$K_\text{f} = \frac{1}{2}mv^2 = (1.60\times10^{-19}\ \text{C})(250\ \text{V}) = 4.00\times10^{-17}\ \text{J} \Rightarrow v = \sqrt{\frac{2(4.00\times10^{-17}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = 9.37\times10^{6}\ \text{ms}$$

The de Broglie wavelength at this speed is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J s}}{(9.11\times10^{-31}\ \text{kg})(9.37\times10^{6}\ \text{m/s})} = 7.77\times10^{-11}\ \text{m}$$

The diameter of the bright spot is the width of the central maximum.

$$w = \frac{2.44\lambda L}{D} = \frac{(2.44)(7.77\times10^{-17}\ \text{m})(1.5\ \text{m})}{33\ \text{nm}} = 8.6\ \text{mm}$$

Assess: This spot seems to have a reasonable diameter.

38.70. **Solve:** The energy of the ultraviolet light photon is

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J s})(3.0\times10^{8}\ \text{m/s})}{70\times10^{-9}\ \text{m}} \times \frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}} = 17.76\ \text{eV}$$

To ionize a hydrogen atom, a minimum energy of 13.60 eV is required. Thus, the kinetic energy of the freed electrons is

$$K = \tfrac{1}{2}mv^2 = E - 13.60\ \text{eV} = 17.76\ \text{eV} - 13.60\ \text{eV} = 4.16\ \text{eV}$$

38.71. **Visualize:** Please refer to Figure 38.13.
Solve: **(a)** From Chapter 20, the rms speed of the sodium atom is

$$v_{\text{rms}} = \sqrt{\frac{3k_{\text{B}}T}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(0.001\ \text{K})}{23\ \text{u}\times1.67\times10^{-27}\ \text{kg/u}}} = 1.04\ \text{m/s} \approx 1.0\ \text{m/s}$$

(b) The diffraction of a wave by a grating obeys the equation $d\sin\theta_m = m\theta,$ where m is the order of the diffraction. The de Broglie wavelength of the atoms is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J s}}{(23\ \text{u}\times1.67\times10^{-27}\ \text{kg/u})(1.04\ \text{m/s})} = 1.66\times10^{-8}\ \text{m}$$

The antinodes in the standing-wave laser beam form the grating. The spacing between adjacent antinodes of a standing wave is

$$d = \frac{\lambda}{2} = 295\ \text{nm} = 2.95\times10^{-7}\ \text{m}$$

Thus, the first order diffraction angle is

$$\theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = \sin^{-1}\frac{1.66\times10^{-8}\ \text{m}}{2.95\times10^{-7}\ \text{m}} = 3.2^\circ$$

(c) The waves traveling to points B and C are each diffracted an angle of 3.2°. From the geometry, the distance between B and C is $2(10\ \text{cm})\tan 3.2^\circ = 1.1\ \text{cm}$.

(d) The atoms exhibit interference when recombined at point D. For interference to occur, each atom must have somehow traveled along *both* paths through the interferometer. So halfway through, *each* atom must have been present both at point B *and* at point C, even though these points are more than 1 cm apart. We're forced to conclude that matter, at the atomic level, does not consist of localized, point-like "particles." Instead, the atoms appear to have the nonlocalized behavior that we previously associated with waves.

38.72. **Solve:** **(a)** The Bohr condition on the electron's angular momentum is $mv_nr_n = n\hbar.$ For cyclotron motion in a magnetic field, the radial component of the force is

$$\frac{mv_n^2}{r_n} = ev_nB \Rightarrow mv_n = eBr_n$$

The Bohr condition becomes

$$(eBr_n)r_n = n\hbar \Rightarrow r_n = \sqrt{\frac{n\hbar}{eB}}$$

(b) The first allowed radius is

$$r_1 = \sqrt{\frac{(1)\hbar}{eB}} = \sqrt{\frac{6.63\times10^{-34}\ \text{J s}}{2\pi(1.6\times10^{-19}\ \text{C})(1.0\ \text{T})}} = 25.7\ \text{nm}$$

Likewise, $r_2 = 36.3\ \text{nm},\ r_3 = 44.5\ \text{nm},$ and $r_4 = 51.4\ \text{nm}.$

(c) For an electron undergoing cyclotron motion in a magnetic field, the kinetic energy is $E_n = \frac{1}{2}mv_n^2.$ Using the Bohr condition in part (a) first for momentum and then for the radius,

$$E_n = \frac{1}{2}\frac{m^2v^2}{m} = \frac{1}{2}\frac{e^2B^2r_n^2}{m} = \frac{eB}{2m}(eBr_n^2) = \frac{eBn\hbar}{2m}$$

From Chapter 30, the cyclotron frequency is

$$f_{\text{cyc}} = \frac{eB}{2\pi m} \Rightarrow E_n = \left(\frac{eB}{2\pi m}\right)(\pi n\hbar) = n(\pi\hbar f_{\text{cyc}})$$